1 The Simplex Algorithm

C is a conjunction of equations;
i, I, j, J, n, m are integers;
a_{ij}, b_i, e_i, d_i are constants;
f, t are linear expressions;
$c_1 \dots c_n$ are equations;
x_i, y_j are variables.

simplex_opt(C, f)
    let C be of the form $c_1 \wedge \cdots \wedge c_n$
    for each $i \in \{1, \dots, n\}$
        let c_i be of the form $x_i = b_i + \sum_{j=1}^{m} a_{ij} y_j$
    endfor
    let f be of the form $e + \sum_{j=1}^{m} d_j y_j$

    % Choose variable y_J to become basic
    if for all $j \in \{1, \dots, m\} d_j \geq 0$ **then**
        return$\langle true, C, f\rangle$
    endif
    choose $J \in \{1, \dots, m\}$ such that $d_J < 0$

    % Choose variable x_I to become non-basic
    if for all $i \in \{1, \dots, n\} a_{iJ} \geq 0$ **then**
        return$\langle false, C, f\rangle$
    endif
    choose $I \in \{1, \dots, n\}$ such that
        $\frac{-b_I}{a_{IJ}} = min\{\frac{-b_i}{a_{iJ}} | a_{iJ} < 0 \text{ and } 1 \leq i \leq n\}$

    $t := \frac{x_I - b_I - \sum_{j=1, j \neq J}^{m} a_{Ij} y_j}{a_{IJ}}$
    $c_I := (y_J = t)$
    replace y_J by t in f
    for each $i \in \{1, \dots, n\}$
        if $i \neq I$ **then** replace y_J by t in c_i **endif**
    endfor
    return simplex_opt$(\wedge_{i=1}^{n} c_i, f)$.